Logistics.

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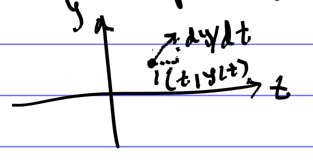
will post notes/problems here.

Format:
- Review basic concepts as needed
- work on those problems in smalls
- discuss solutions as a class
- repeat

Suppose you are given the following diff eq.

$$\frac{dy}{dt} = f(t,y)$$

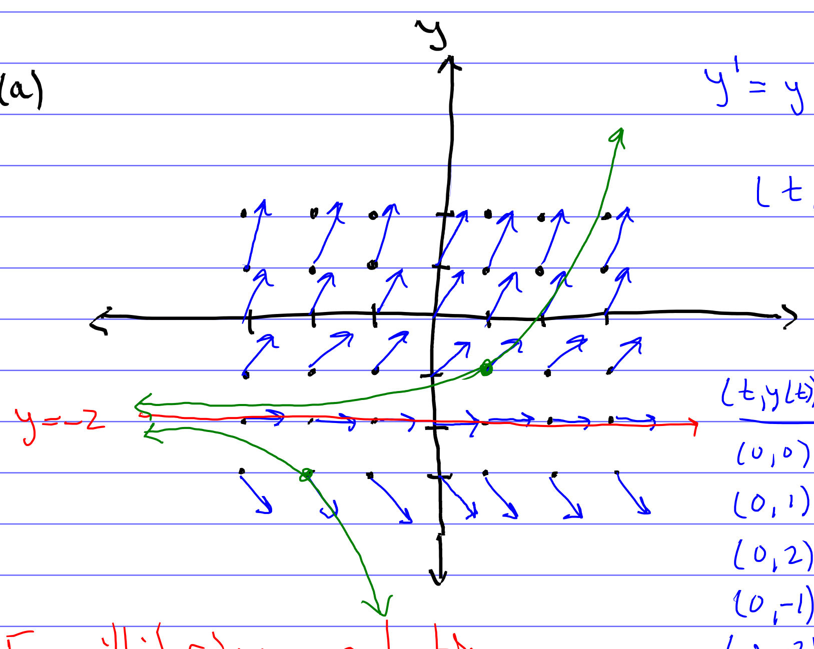
one can investigate solutions $y(t)$ by drawing a direction field. Procedure: for each pt $(t, y(t))$ in the plane, draw an arrow w/ initial pt $(t, y(t))$ and slope $\frac{dy}{dt}$



1. Draw a direction field and determine the behaviour of y as $t \to \infty$.

(a) $y' = y + 2$
(b) $y' = t + y$

(a)



$y' = y + 2$

$(t, y(t))$

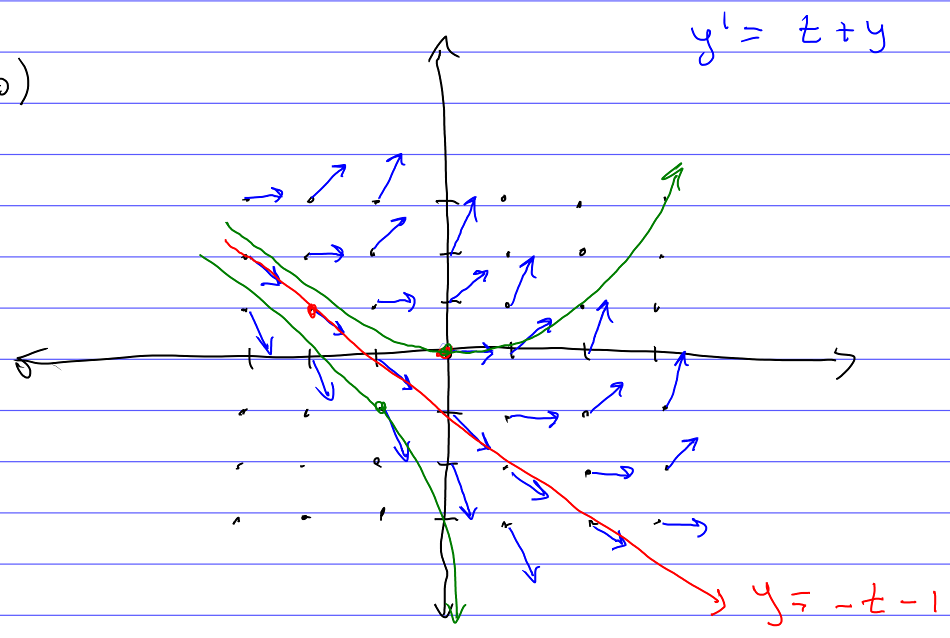
$(t,y(t))$	y'
$(0,0)$	2
$(0,1)$	3
$(0,2)$	4
$(0,-1)$	1
$(0,-2)$	0

Equilibrium solution $y=-2$

If $y > -2$, then $y \to \infty$ as $t \to \infty$
If $y < -2$, then $y \to -\infty$ as $t \to \infty$.

(b)

$y' = t + y$



$y \to \infty$ as $t \to \infty$ or $y \to -\infty$ as $t \to \infty$, depending on whether the init. condition is above or below $y = -t-1$.

Recall: diff. eq. of the form

$$\frac{dy}{dt} = ay - b = a\left(y - \frac{b}{a}\right)$$

can be solved using integration:

$$\Rightarrow \quad \frac{dy/dt}{y - \frac{b}{a}} = a$$

integrate
$\Rightarrow$

$$\ln\left|y - \frac{b}{a}\right| = \int \frac{dy/dt}{y - \frac{b}{a}}\,dt = \int a\,dt$$
$$= at + C$$

solve for y
$\Rightarrow$

$$\boxed{\begin{aligned} y &= e^{at+C} + \frac{b}{a} \\ &= \mathcal{C}e^{at} + \frac{b}{a} \end{aligned}} \qquad (\mathcal{C} = e^{C})$$

2. Solve each initial value problem (IVP) and plot solutions for several values of y_0.

(a) $\frac{dy}{dt} = y - 5$, $y(0) = y_0$
(b) $\frac{dy}{dt} = 2y - 5$, $y(0) = y_0$

(a) $\frac{dy/dt}{y-5} = 1 \Rightarrow \ln|y-5| = t + c$

$\Rightarrow \boxed{y = ce^{t} + 5}$

Using init. cond.

$y_0 = y(0) = c + 5$

$\Rightarrow \boxed{c = y_0 - 5}$

y_0	c
-2	-7
-1	-6
0	-5
1	-4
2	-3